Simulation-based Optimization Methods for High-dimensional Urban Mobility Problems

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Mobility systems are quickly evolving:

- Connected (V2V, V2I, IoT): demand & supply interactions are complex
- Local changes can have instantaneous large-scale impacts
- Real-time responsive demand & supply.
- Generally, they are becoming more intricate

- What is the role of analytical models, from OR and control theory, in this era?
- How can our analytical models help us search in high-dimensional spaces?

A biased response …

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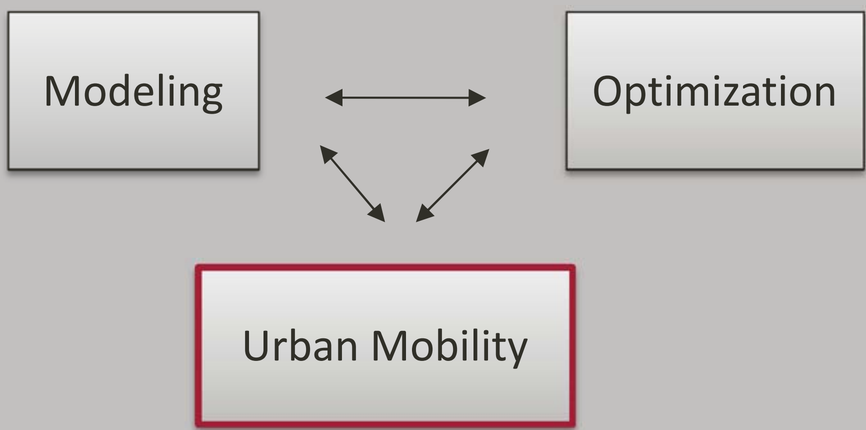
My bias



- Private and public stakeholders
- Problems: signal control, congestion pricing, autonomous mobility, car-sharing, calibration
- Goal: design practical algorithms for stakeholders, computational efficiency

Travel time reliability	2019, *Transp. Science*
Dynamic problems	2016, *Transp. Science*
Energy efficiency	2015, *Transp. Science*
Large-scale	2015, *Transp. Science*
Emissions	2015, *Transp. Part B*

Simulation-based optimization

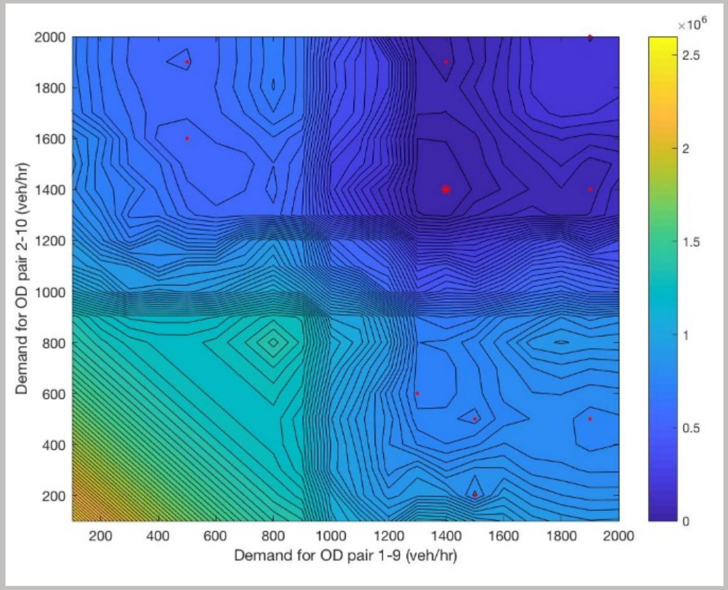
$$\min_{x \in \Omega} f(x) = E[F(x)]$$

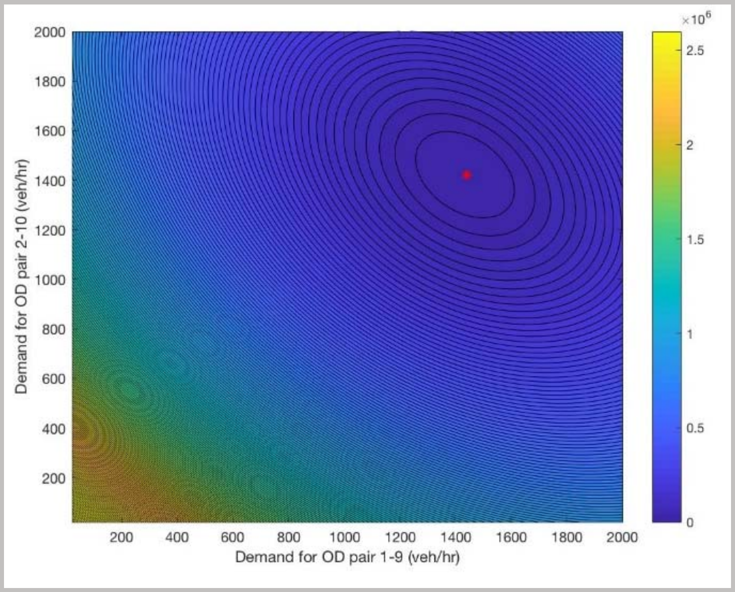
- Challenging problem
  - Objective function
    - No closed-form expression
    - Unknown mathematical properties (e.g., convexity)
    - Computationally costly to evaluate
  - High-dimensional problems (1000-10,000 variables)
  - Most common approach: use of general-purpose algorithms (e.g., SPSA)

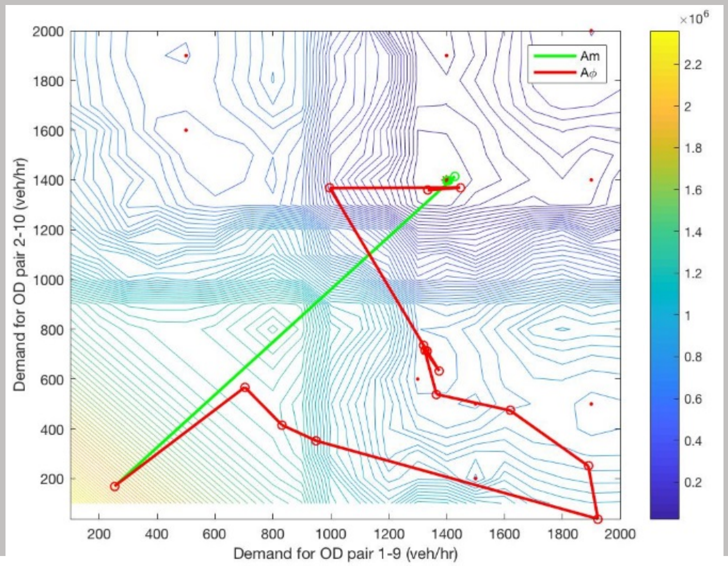
- Use of analytical models to enable general-purpose algorithms to become scalable and computationally efficient

Two-dimensional example

$$\min_{x \in \Omega} f(x) = E[F(x)] \quad \leftrightarrow \quad \min_{x \in \Omega \cap \Psi_k} m_k(x;\beta) = \beta_0 \, A(x) + \Phi(x;\beta)$$







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Continuous Problems

- Suitable for a broad family of transportation problems
  - Signal control, congestion pricing, OD calibration
  - Non-convex : energy consumption, emissions
  - High-dimensional : 16K decision variables
  - Efficiency: ~15-100 simulation runs
  - Large-scale networks: over 24,000 links
  - Dynamic, real-time

What happens in discrete space?

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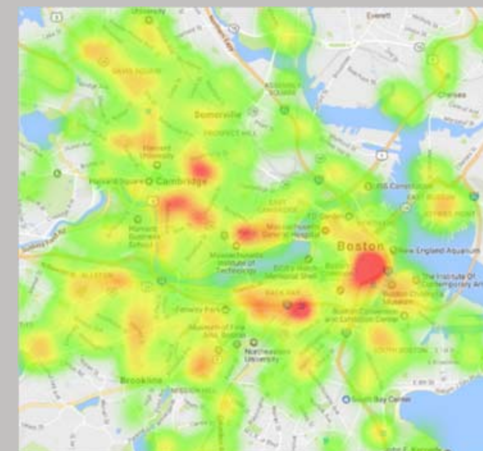
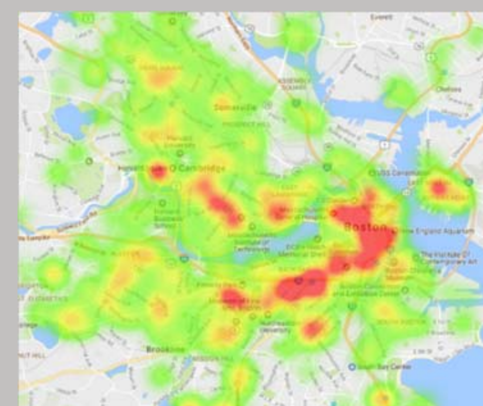
Integrated On-demand Mobility Services

- Boston, Chicago NYC, San Francisco, Toronto,
- Zipcar data
  - Station data: location, space capacity, costs
  - Reservation data: vehicle, creation time, start time, end time, revenue
- Used disaggregate reservation data to
  - Estimate demand distribution
  - To “simulate” the reservation process to estimate the expected revenue
  - Low-parametric simulator designed in collaboration with stakeholders





Spatially assign vehicles such as to maximize expected profit

$$\max_{x} \quad g(x;q_1) = E[R(x;q_1)] - \sum_{i \in \mathcal{I}} c_i x_i \qquad (1)$$

$$\sum_{i \in \mathcal{I}} x_i \leq X \qquad (2)$$

$$x_i \leq N^i \quad \forall i \in \mathcal{I} \qquad (3)$$

$$x_i \in \mathbb{Z}_+ \quad \forall i \in \mathcal{I}. \qquad (4)$$

Metamodel Problem

$$\max_x \quad g(x;q_1) = E[R(x;q_1)] - \sum_{i\in\mathcal{I}} c_i x_i$$
$$x \in \mathcal{F}$$

$$\max_x \quad m_k(x;\beta_k,q_2) = \beta_{k,0} g_A(x,z;q_2) + \phi(x;\beta_k)$$
$$g_A(x,z) = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{I}_i} \sum_{t\in\mathcal{T}} \sum_{l\in\mathcal{L}} p^{ij} r_{tl} z_{tl}^{ij} - \sum_{i\in\mathcal{I}} c_i x_i,$$
$$x \in \mathcal{F}$$
$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ji} = z_{tl}^{i} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}$$
$$\sum_{j\in\mathcal{I}_i} z_{tl}^{ij} \le d_{tl}^{i} \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}$$
$$\sum_{l\in\mathcal{L}} z_{tl}^{i} + \sum_{l\in\mathcal{L}} \sum_{t'\in\mathcal{T}_1(t,l)} z_{t'l}^{i} \le x_i \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}$$
$$z_{tl}^{i} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}$$
$$z_{tl}^{ij} \in \mathbb{R}_+ \qquad \forall i\in\mathcal{I}, \forall j\in\mathcal{I}_i, \forall t\in\mathcal{T}, \forall l\in\mathcal{L}$$

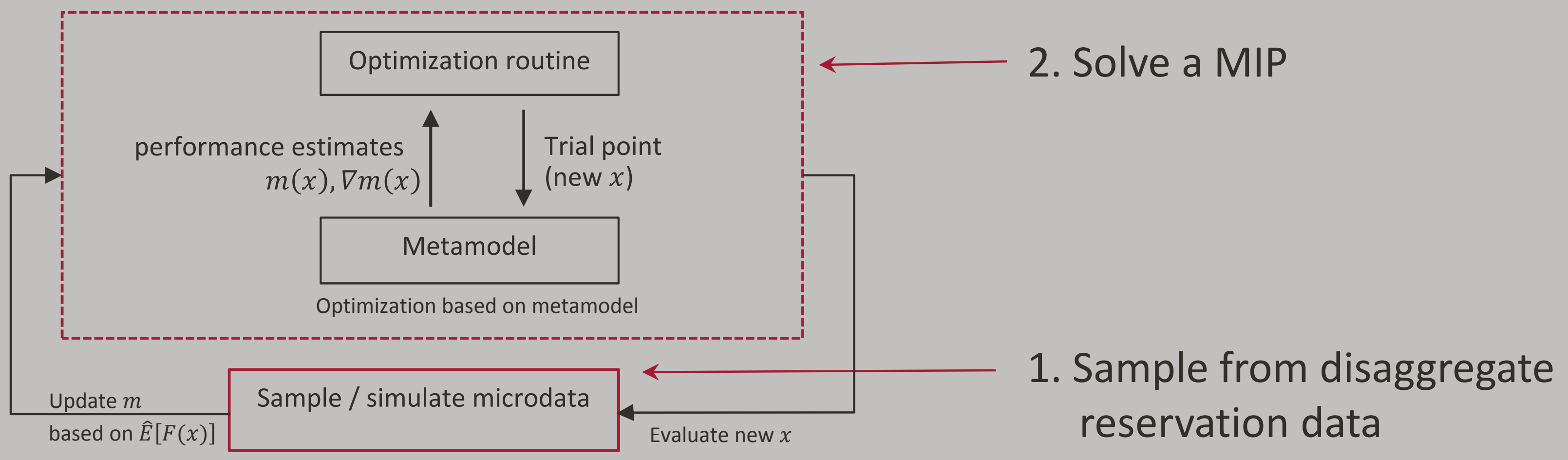
Metamodel

Customer flow conservation

Demand constraint

Supply constraint

Metamodel Approach



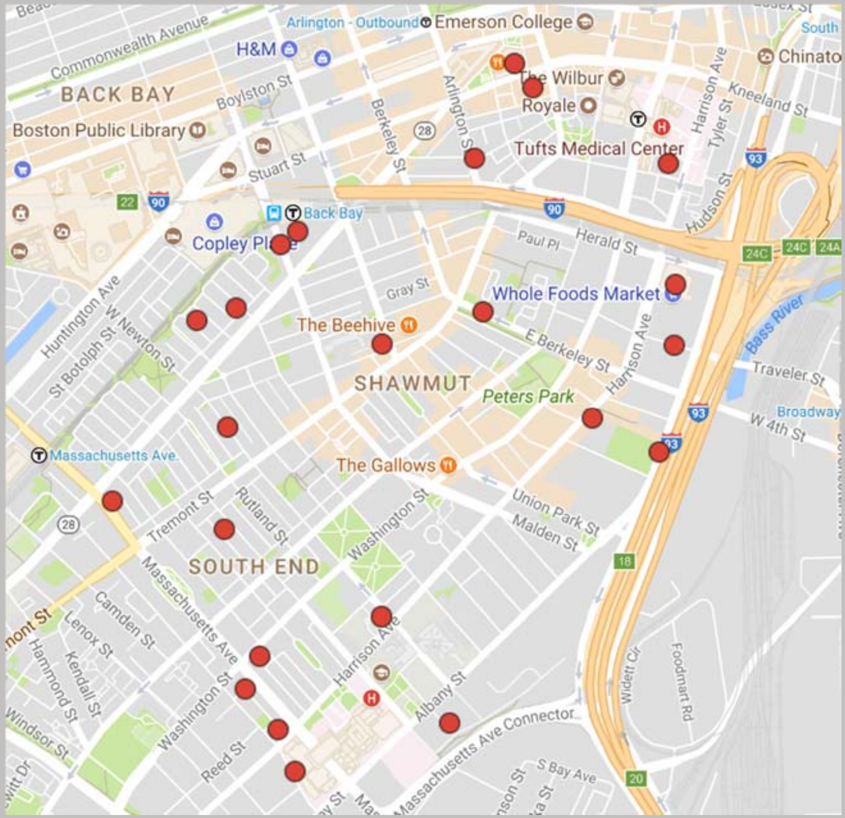
- Algorithm: extension of AHA of Xu, Nelson and Hong (2013) “An adaptive hyperbox algorithm for high-dimensional discrete optimization via simulation problems”, *INFORMS Journal on Computing*

- At every iteration of the algorithm:
  1. Sample/simulate
  2. Solve an analytical MIP problem
  3. Sample solution from (2) and sample other points (e.g., random)
  4. Use the simulation observations to fit the parameters of the metamodel

Downtown Boston Car-sharing

- Boston South End
- 23 stations
- Total fleet size: 101 cars
- One week in July 2014

- Stop algorithm after 25 iterations
- Simulate 10 points per iteration
- Evaluation under different demand scenarios



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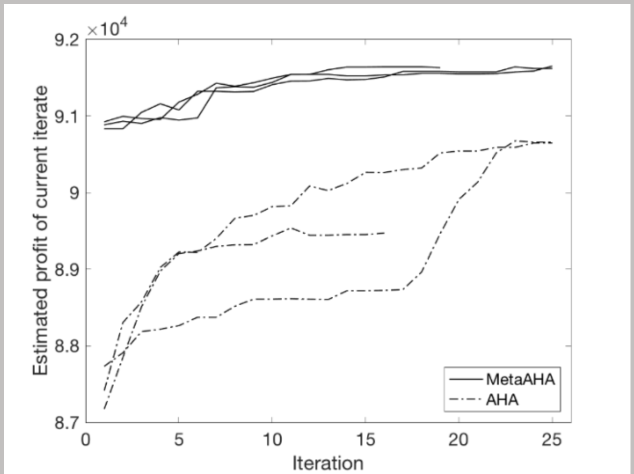
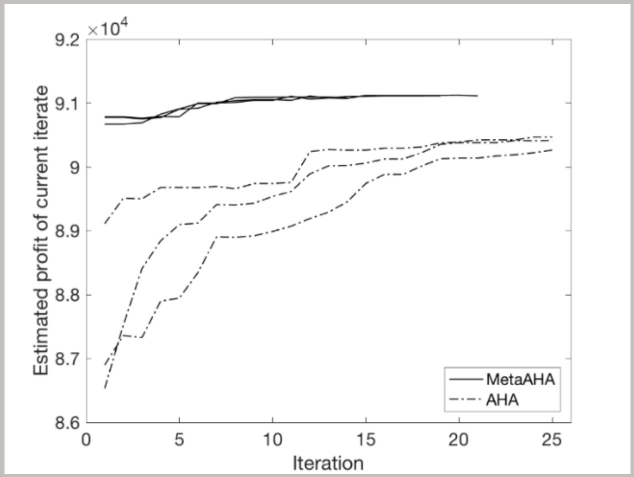
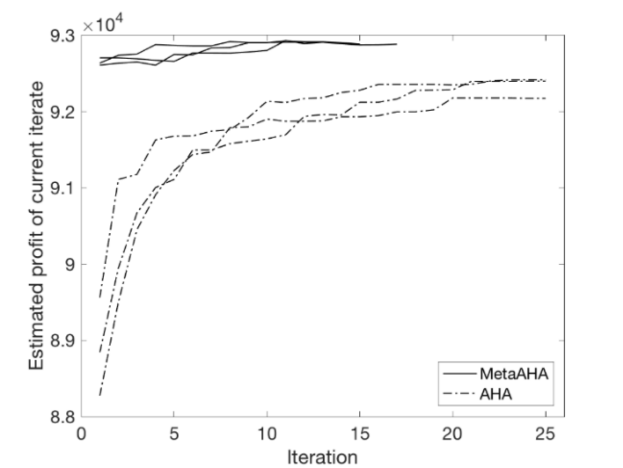
Downtown Boston Car-sharing

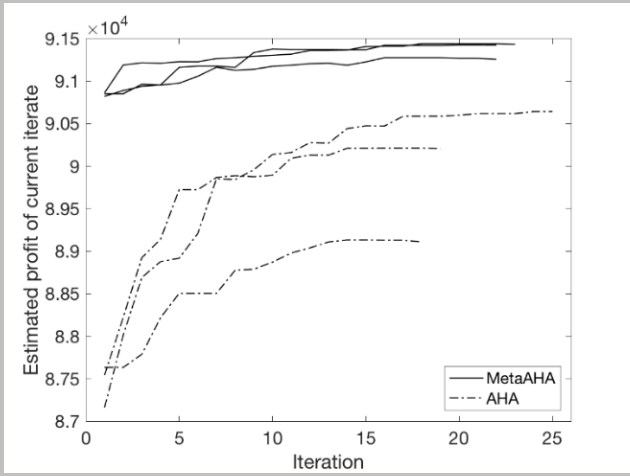
Comparison versus AHA

- Improved performance from the very first iteration
- Performance is robust to the quality of the initial solutions









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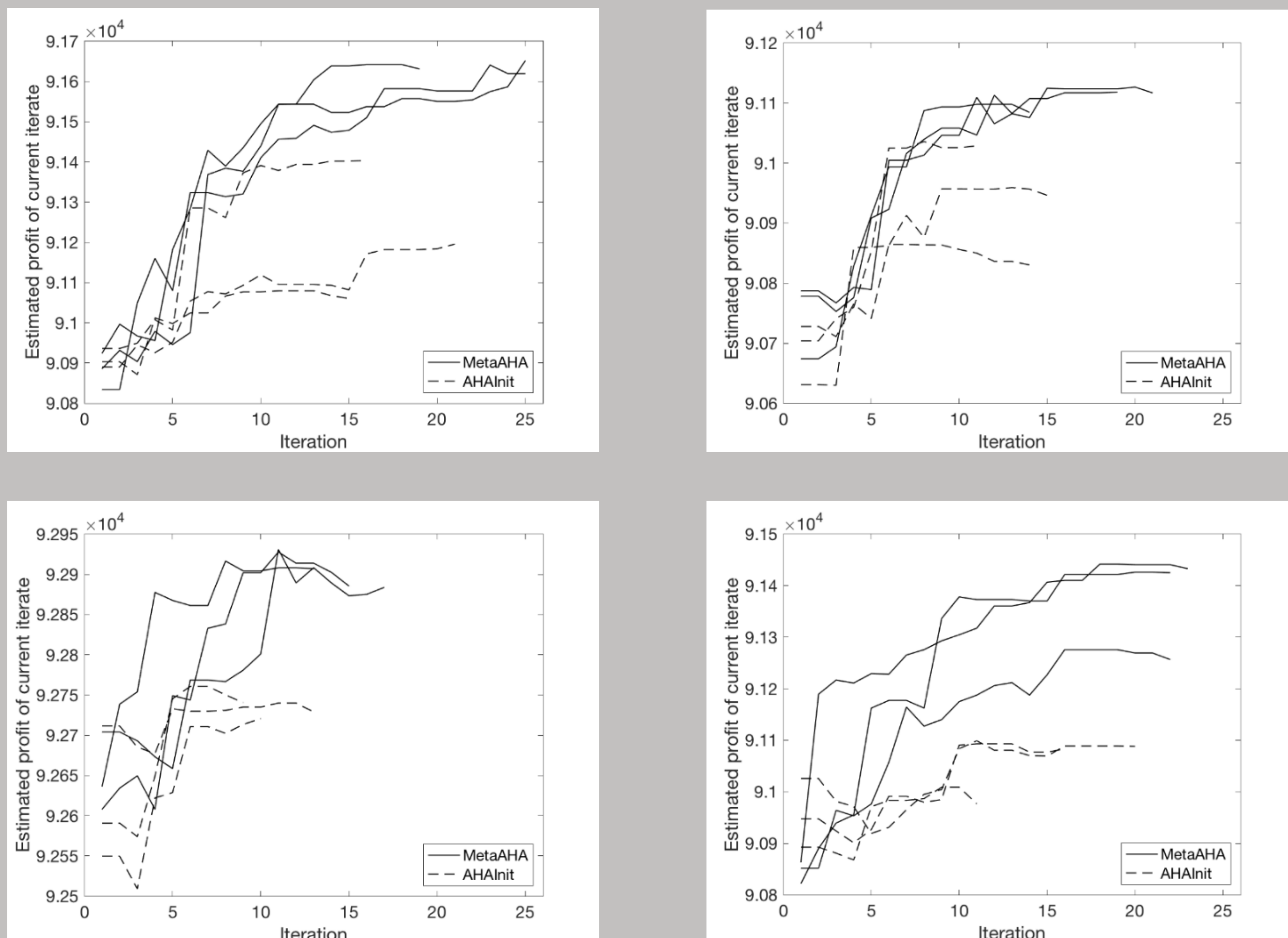
Downtown Boston Car-sharing

Comparison versus AHAInit

- There is an added value in using the MIP information across iterations



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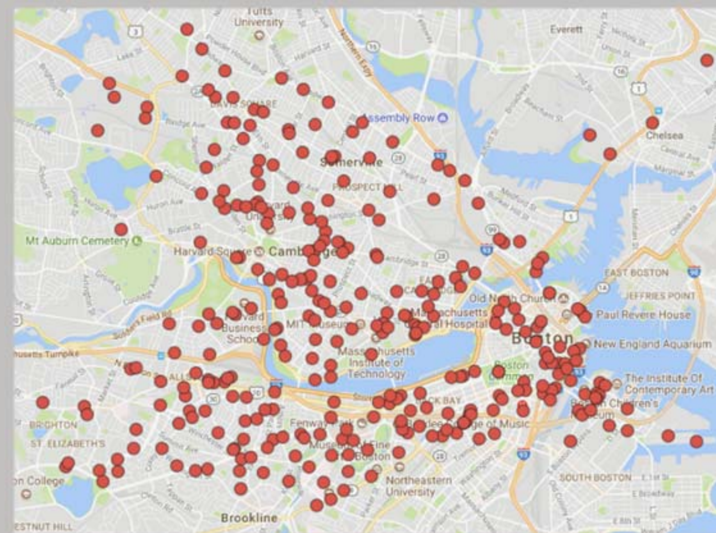
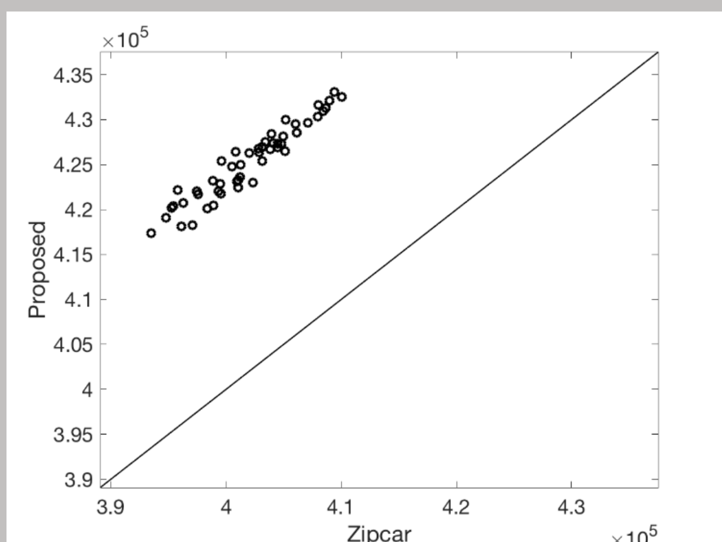
Metro Boston Car-sharing

- Comparison versus field deployed solution
- Larger Boston metropolitan area (23 zipcodes)
- 315 stations, fleet size: 894 cars
- One week in July 2014

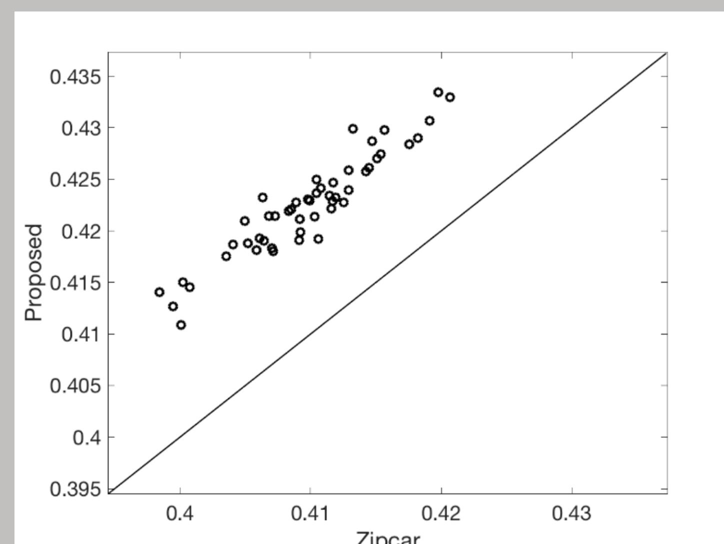
- Stop algorithm after 40 iterations
- Simulate 70 points per iteration
- Evaluation under different demand scenarios



Profit



Utilization



Insights

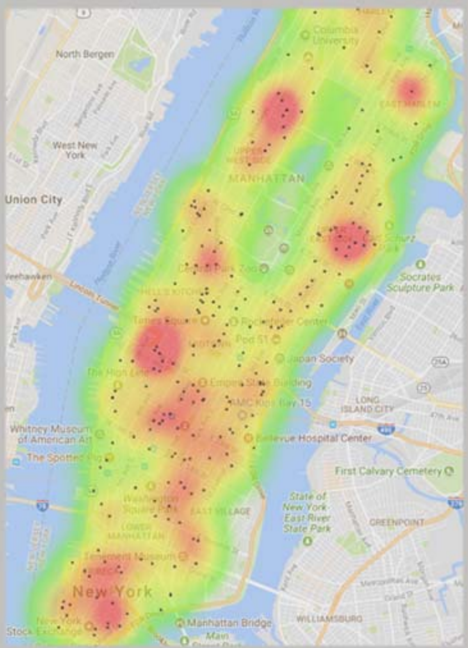
- Information from an analytical MIP enhances the scalability and the computational efficiency of general-purpose discrete simulation-based optimization algorithms
- There is abundant analytical literature (IP/MIP) we can build upon

What is the role of simple analytical models in this era?

- Leave the realism to the data
- Devise creative ways of combining analytical models with more realistic/data-driven approaches
- Analytical models can provide problem structure to general-purpose black-box methods
- Search high-dimensional spaces, preserve asymptotic guarantees + achieve computational efficiency

Ongoing work:

- Use of MIP as a sampling distribution
  - High-dimensional sampling techniques
  - Scalable Bayesian optimization: GPs + analytical models
- Algorithms to optimize both profit and transportation accessibility
- Use of search data for demand estimation



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Great Team

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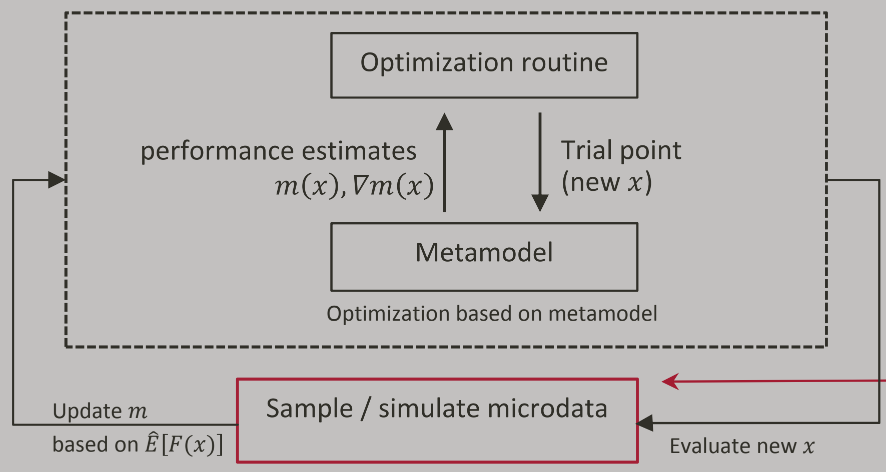
- Prof. Bilge Atasoy (TU Delft)
- Prof. Cynthia Barnhart (MIT)
- Prof. Gunnar Flötteröd (VTI)
- Prof. Vincenzo Punzo (Uni. Napoli)
- Prof. Bruno Santos (TU Delft)

Questions ?

Discrete & Data-driven Simulation-Optimization

$$\min_{x \in \Omega} f(x) = E[F(x)]$$



1. Sample from high-resolution mobility data

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$$\min_{\beta_{kt}} \sum_{d \in \mathcal{S}_k} \{w_k(d)\,(\hat{g}_t(d) - m_{kt}(d_t;\beta_{kt}))\}^2 + w_0^2 \left((\beta_{kt0} - 1)^2 + \beta_{kt1}^2 + \sum_{z=2}^{Z+1} \beta_{ktz}^2 \right)$$

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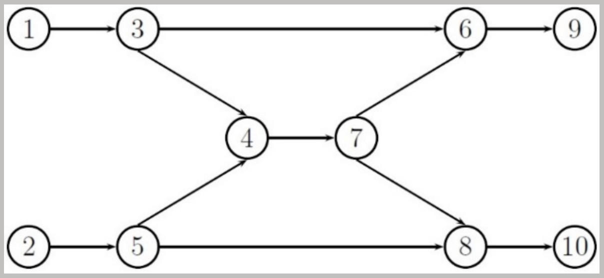
2D example

$$\min_{d} f(d) = \sum_{i \in \mathcal{I}} \left(y_i - E[F_i(d, u_1; u_2)]\right)^2 + \delta \sum_{z \in \mathcal{Z}} \left(\tilde{d}_z - d_z\right)^2$$

$$d \geq 0$$



$$\min_{d_z} \sum_{i \in \mathcal{I}} \left(y_i - m_{i,k}(d_z; \beta_{i,k})\right)^2 + \delta \sum_{z \in \mathcal{Z}} \left(d_z - \hat{d}_z\right)^2$$

$$d_z \geq 0$$

$$h(d_z, \tilde{v}; \tilde{q}) = 0$$

$$m_{i,k}(d_z; \beta_{i,k}) = \beta_{i,k,0}\lambda_i(d_z) + \phi(d_z; \beta_{i,k})$$

$$\lambda_i = \sum_{z \in \mathcal{Z}} \sum_{s \in \mathcal{T}_i} d_z \tilde{p}_{zs} + \sum_{j \in \mathcal{Q}} p_{ji}\lambda_j$$

- For a network with n links: system of n linear equations
- Complexity scales linearly with the number of links
- Tractable & scalable

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