HOW MANY SMART CARS DOES IT TAKE TO MAKE A SMART TRAFFIC NETWORK?

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WHY CAN’T WE IMPROVE TRAFFIC…

… EVEN IF WE KNOW THE ACHIEVABLE OPTIMUM IN A TRAFFIC NETWORK ???

Because:

- **Not enough controls** (traffic lights, tolls, speed fines)
  → No chance to unleash the power of feedback!

- **Not knowing other drivers’ behavior** leads to poor decisions (a simple game-theoretic fact)
  → Drivers seek individual (**selfish**) optimum, not system-wide (**social**) optimum

**PRICE OF ANARCHY (POA)**
GAME-CHANGING OPPORTUNITY: CONNECTED AUTONOMOUS VEHICLES (CAVs)

FROM (SELFISH) “DRIVER OPTIMAL” TO (SOCIAL) “SYSTEM OPTIMAL” TRAFFIC CONTROL

THE “INTERNET OF CARS”

NO TRAFFIC LIGHTS, NEVER STOP…
A DECENTRALIZED OPTIMAL CONTROL FRAMEWORK FOR CAVs

NO TRAFFIC LIGHTS, NEVER STOP...
CONFLICT AREAS
- COOPERATIVE CONTROL OPPORTUNITIES

Merge: roundabout
Intersection: with/without signal
Merge and pass: lane change maneuver
Merge: on-ramp
CONTROL ZONES

CONTROL ZONE (CZ): Vehicles can cooperate to achieve desirable performance

Minimize Travel Time through CZ

Minimize Energy through CZ

Maximize Passenger Comfort

Guarantee Safety
The problem is to minimize the following objective function:

$$\min_{u_i(t)} w_1(t_i^m - t_i^0) + \frac{1}{2} \int_{t_i^0}^{t_i^m} [w_2 u_i^2(t) + w_3 J_i^2(t)] dt$$

subject to:

1. CAV dynamics
2. Speed/Acceleration constraints
3. Safety constraints
4. Given $t_i^0$, $x_i(t_i^0)$, $v_i(t_i^0)$, $x_i(t_i^m)$
5. $\sum_{i=1}^{3} w_i = 1$, $w_i \in [0,1]$

...for ANY CZ defined in the traffic network
THE INTERSECTION MODEL

CAV dynamics:

\[
\begin{align*}
\dot{p}_i &= v_i(t) \\
\dot{v}_i &= u_i(t) \\
t &\in [t_{i}^0, t_{i}^f]
\end{align*}
\]

\(t_{i}^0\) : Enters Control Zone (CZ)

\(t_{i}^f\) : Exits Merging Zone (MZ)

Speed, Acceleration constraints:

\[
\begin{align*}
0 &\leq v_{\text{min}} \leq v_i(t) \leq v_{\text{max}} \\
0 &\leq u_{\text{min}} \leq u_i(t) \leq u_{\text{max}}
\end{align*}
\]
CAV \( i \) MINIMIZATION PROBLEM

\[
\min_{u_i(t)} \gamma(t_i^m - t_i^0) + \int_{t_i^0}^{t_i^m} \frac{1}{2} u_i^2(t) dt
\]

subject to:

1. CAV dynamics
2. Speed/Acceleration constraints
3. Order constraints: \( t_i^m \geq t_{i-1}^m \)
4. Rear-end safety constraint
5. Lateral collision avoidance constraint

\[
p_i(t_i^0) = 0, \quad p_i(t_i^m) = L, \quad \text{given: } t_i^0, \; v_i(t_i^0)
\]

Each CAV minimizes TRAVEL TIME + ENERGY COST FUNCTIONAL
SOLUTION – NO ACTIVE CONSTRAINTS

\[ u_i^*(t) = a_i t + b_i \]

\[ v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i \]

\[ p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i \]

Coefficients and optimal merging time obtained from:

\[
\frac{1}{6} a_i \cdot (t_i^0)^3 + \frac{1}{2} b_i \cdot (t_i^0)^2 + c_i t_i^0 + d_i = 0
\]

\[
\frac{1}{2} a_i \cdot (t_i^0)^2 + b_i t_i^0 + c_i = v_i^0
\]

\[
\frac{1}{6} a_i \cdot (t_i^m)^3 + \frac{1}{2} b_i \cdot (t_i^m)^2 + c_i t_i^m + d_i = L
\]

\[
a_i t_i^m + b_i = 0
\]

\[
\gamma - \frac{1}{2} b_i^2 + a_i c_i = 0
\]

THEOREM:
The optimal control is \( u_i^*(t) \geq 0 \) and monotonically non-increasing.
When constraints are active:

Solution is of the same form and still analytically tractable

- Malikopoulos, Cassandras, and Zhang, *Automatica*, 2018
- Zhang and Cassandras, *Automatica*, 2019 (subm.)
WHO NEEDS TRAFFIC LIGHTS?

One of the worst-designed double intersections ever…
(BU Bridge – Commonwealth Ave, Boston, MA)
EXAMPLE

WIN-WIN!

+ fewer harmful emissions

**Fuel Consumption (434 vehicles)**

- 46.63% improvement

**Average Travel Time**

- 30.89% improvement

![Graphs showing fuel consumption and average travel time improvements.](image)

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WHAT HAPPENS IN MIXED TRAFFIC?

- CAVs
- Non-CAVs
MIXED TRAFFIC - CAV BEHAVIOR

\[
\min \frac{1}{2} \int_{t_i}^{t_i^m} [u_i^2(t) + \gamma(s_i(t) - \delta)^2] dt
\]

subject to:
1. CAV dynamics
2. Speed/Acceleration constraints
   \[ t_i^m, p_i(t_i^m) = L, \text{ given: } t_i^0, p_i(t_i^0), v_i(t_i^0) \]
MIXED TRAFFIC – NON-CAV BEHAVIOR

- Car-following behavior: The **Wiedemann Model** [Wiedemann, 1974]
- Collision avoidance model in MZ through Conflict Areas.
Traffic Flow Rate = 700 veh/(hourlane)
ENERGY IMPACT OF CAV PENETRATION

NOTE: Impact depends on Traffic Flow Rate!
ENERGY IMPACT OF CAV PENETRATION

NOTE: Impact depends on CAV and Non-CAV behavior models
CAV PENETRATION IMPACT IN TRAFFIC ROUTING

Eastern Mass.
13,000+ road segments

LINK $a$  
FLOW $x_a$  
COST FUNCTION $t_a(x_a)$

USER-CENTRIC (selfish) control - Non-CAVs: $x_a^{user}$ is the equilibrium flow

SYSTEM-CENTRIC (social) control - CAVs: $x_a^{social}$ is the equilibrium flow

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DO NON-CAVs BENEFIT FROM CAV PENETRATION?

Non-CAVs (selfish users) benefit from the addition of CAVs!

**INTUITION:** CAVs improve resource allocation for everyone, e.g., they decongest a link so that Non-CAVs still using this link benefit.
DO NON-CAVs BENEFIT FROM CAV PENETRATION?

What incentive does a selfish user have to switch to a cooperative game setting (i.e., get a CAV)???
When it is optimal for CAVs to **decelerate**, Non-CAVs are induced to act optimally (natural platoons formed)

When it is optimal for CAVs to **accelerate**, Non-CAVs become obstacles inducing sub-optimality

Incentives for Non-CAVs to convert to CAVs?

*Is Shared Mobility **On-Demand** the long-term answer?*
*(typical car utilization is 4%...)*