HOW MANY SMART CARS DOES IT TAKE TO MAKE A SMART TRAFFIC NETWORK?

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… EVEN IF WE KNOW THE ACHIEVABLE OPTIMUM IN A TRAFFIC NETWORK ???

Because:

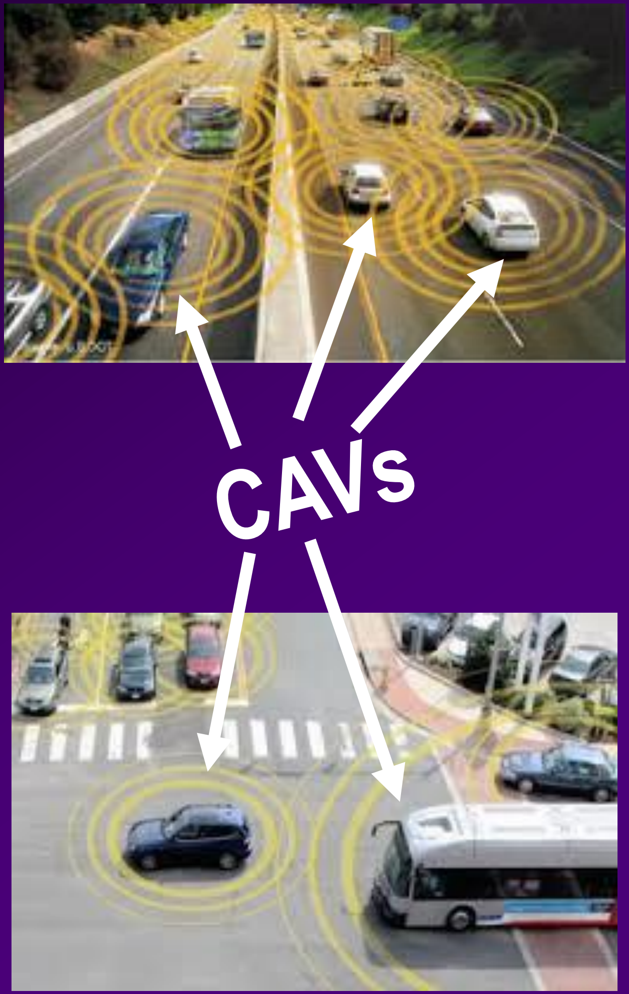
- **Not enough controls** (traffic lights, tolls, speed fines)

  → No chance to unleash the power of feedback!

- **Not knowing other drivers' behavior** leads to poor decisions (a simple game-theoretic fact)

  → Drivers seek individual (**selfish**) optimum, not system-wide (**social**) optimum } *PRICE OF ANARCHY* (POA)

GAME-CHANGING OPPORTUNITY:
CONNECTED AUTONOMOUS VEHICLES (CAVs)



FROM (SELFISH) “DRIVER OPTIMAL” TO (SOCIAL) “SYSTEM OPTIMAL” TRAFFIC CONTROL

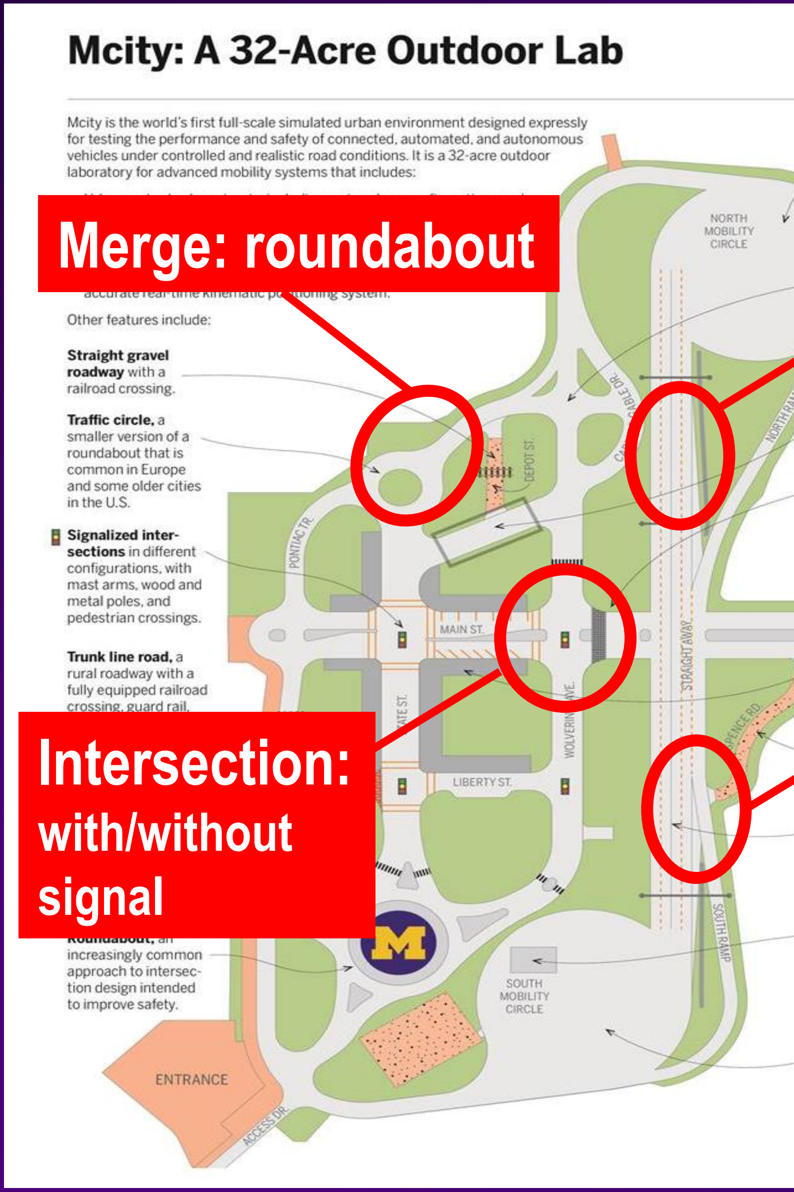
THE “INTERNET OF **CARS**”

NO TRAFFIC LIGHTS, NEVER STOP…

A DECENTRALIZED OPTIMAL CONTROL FRAMEWORK FOR CAVs

NO TRAFFIC LIGHTS, NEVER STOP…

CONFLICT AREAS
- COOPERATIVE CONTROL OPPORTUNITIES



Merge: roundabout

Merge and pass:
lane change maneuver

Intersection:
with/without
signal

Merge: on-ramp

CONTROL ZONES

CONTROL ZONE (CZ):
Vehicles can cooperate to achieve desirable performance

Minimize Travel Time through CZ

+

Minimize Energy through CZ

+

Maximize Pssenger Comfort

+

Guarantee Safety

DECENTRALIZED OPTIMAL CONTROL PROBLEM

Travel Time — Energy — Comfort

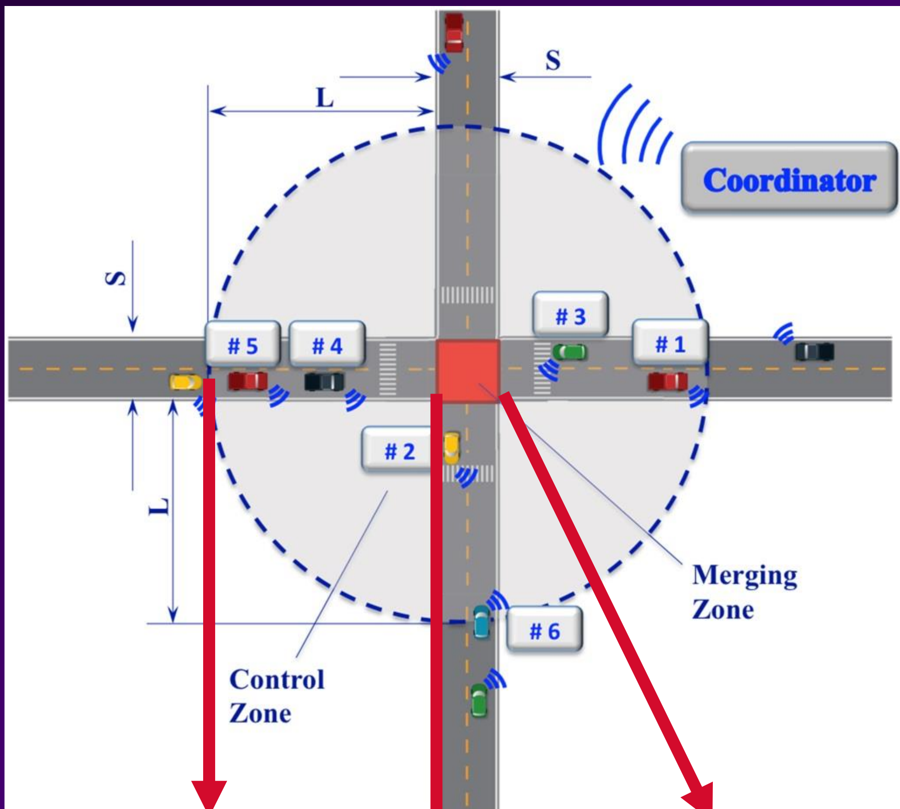
$$\min_{u_i(t)} \; w_1(t_i^m - t_i^0) + \frac{1}{2}\int_{t_i^0}^{t_i^m} [w_2 u_i^2(t) + w_3 J_i^2(t)]dt$$

subject to :

1. CAV dynamics
2. Speed/Acceleration constraints
3. Safety constraints
4. Given $t_i^0, \; x_i(t_i^0), \; v_i(t_i^0), \; x_i(t_i^m)$
5. $\sum_{i=1}^{3} w_i = 1, \; w_i \in [0,1]$

…for ANY CZ defined in the traffic network

THE INTERSECTION MODEL



Enters CZ

Enters MZ at time t_i^m

Exits MZ

CAV dynamics:

$$\dot{p}_i = v_i(t)$$
$$\dot{v}_i = u_i(t)$$
$$t \in [t_i^0, t_i^f]$$

t_i^0 : Enters Control Zone (CZ)

t_i^f : Exits Merging Zone (MZ)

Speed, Acceleration constraints:

$$u_{\min} \le u_i(t) \le u_{\max}$$
$$0 \le v_{\min} \le v_i(t) \le v_{\max}$$

CAV i MINIMIZATION PROBLEM

$$\min_{u_i(t)} \quad \gamma(t_i^m - t_i^0) + \int_{t_i^0}^{t_i^m} \frac{1}{2} u_i^2(t)\,dt$$

subject to: 1. CAV dynamics

2. Speed/Acceleration constraints

3. Order constraints: $t_i^m \geq t_{i-1}^m$

4. Rear-end safety constraint

5. Lateral collision avoidance constraint

$$p_i(t_i^0) = 0, \quad p_i(t_i^m) = L, \quad \text{given}: \; t_i^0, \; v_i(t_i^0)$$

Each CAV minimizes TRAVEL TIME + ENERGY COST FUNCTIONAL

SOLUTION – *NO ACTIVE CONSTRAINTS*

$$u_i^*(t) = a_i t + b_i$$

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i$$

$$p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i$$

Coefficients and optimal merging time obtained from:

$$\begin{aligned}
\frac{1}{6} a_i \cdot (t_i^0)^3 + \frac{1}{2} b_i \cdot (t_i^0)^2 + c_i t_i^0 + d_i &= 0 \\
\frac{1}{2} a_i \cdot (t_i^0)^2 + b_i t_i^0 + c_i &= v_i^0 \\
\frac{1}{6} a_i \cdot (t_i^m)^3 + \frac{1}{2} b_i \cdot (t_i^m)^2 + c_i t_i^m + d_i &= L \\
a_i t_i^m + b_i &= 0 \\
\gamma - \frac{1}{2} b_i^2 + a_i c_i &= 0
\end{aligned}$$

THEOREM:
The optimal control is $u_i^*(t) \geq 0$ and monotonically non-increasing

SOLUTION – *MULTIPLE CONSTRAINTS ACTIVE*

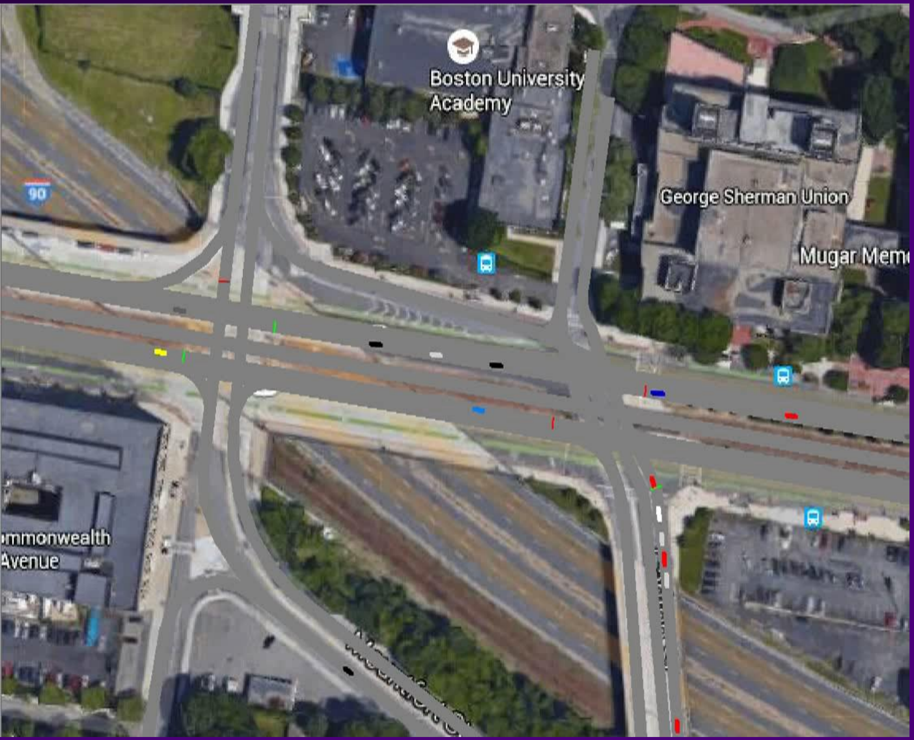
When constraints are active:

Solution is of the same form and still analytically tractable

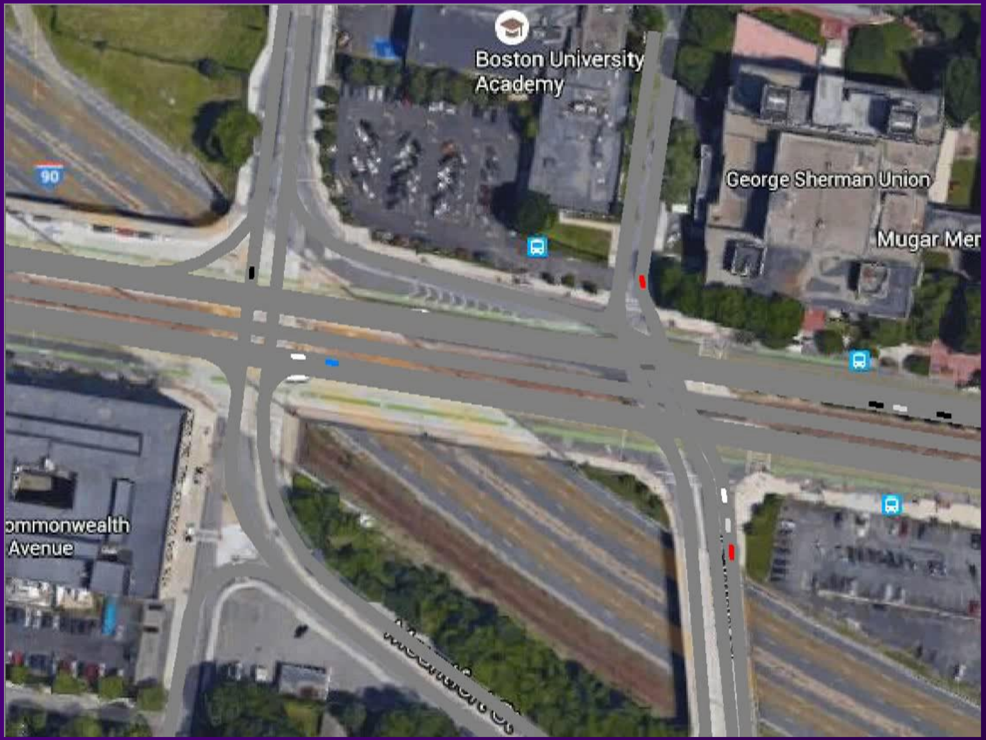
- Malikopoulos, Cassandras, and Zhang, *Automatica, 2018*
- Zhang and Cassandras, *Automatica, 2019 (subm.)*

WHO NEEDS TRAFFIC LIGHTS?

With traffic lights



With decentralized control of CAVs



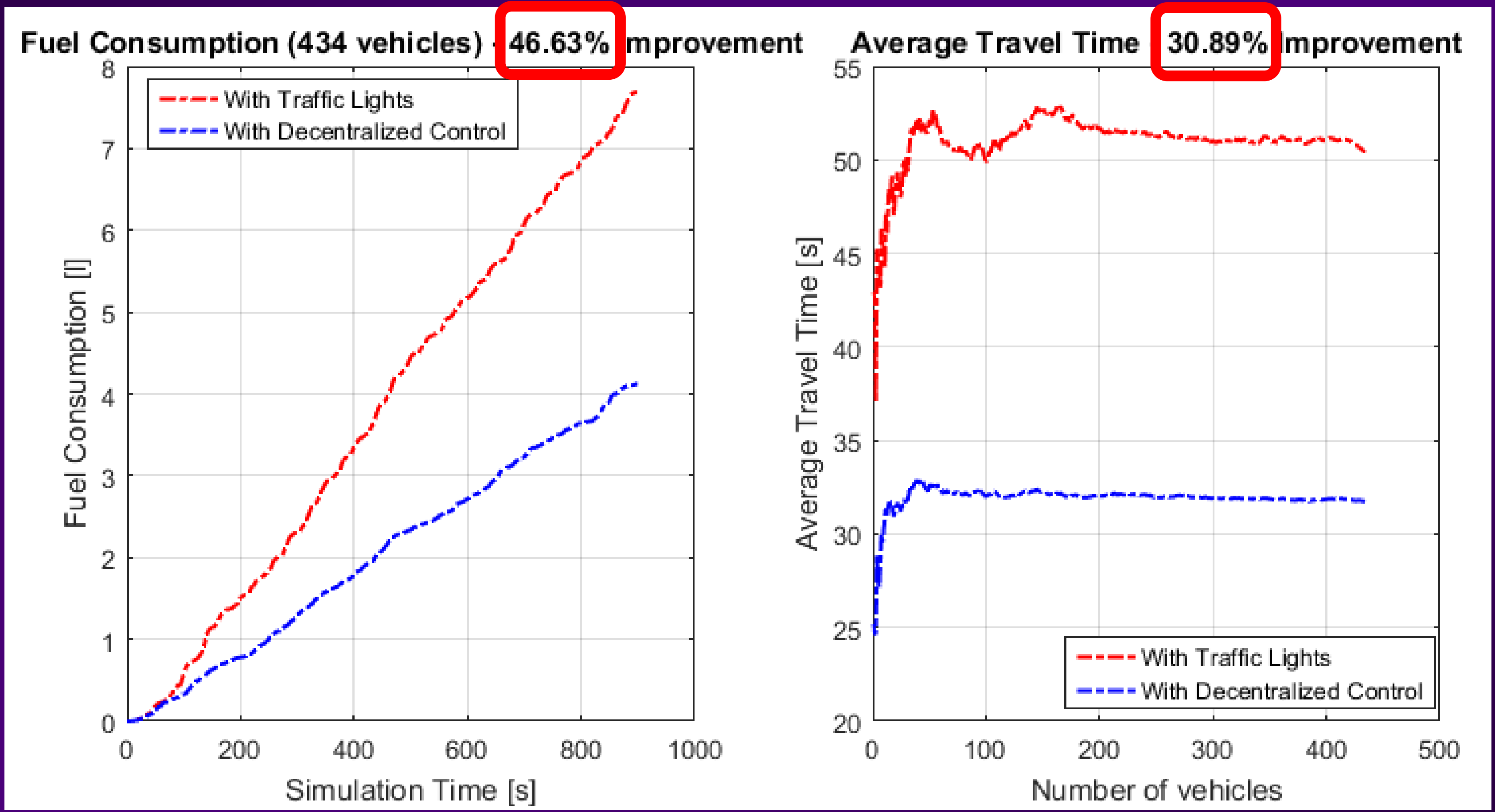
One of the worst-designed double intersections ever…

(BU Bridge – Commonwealth Ave, Boston, MA)

EXAMPLE

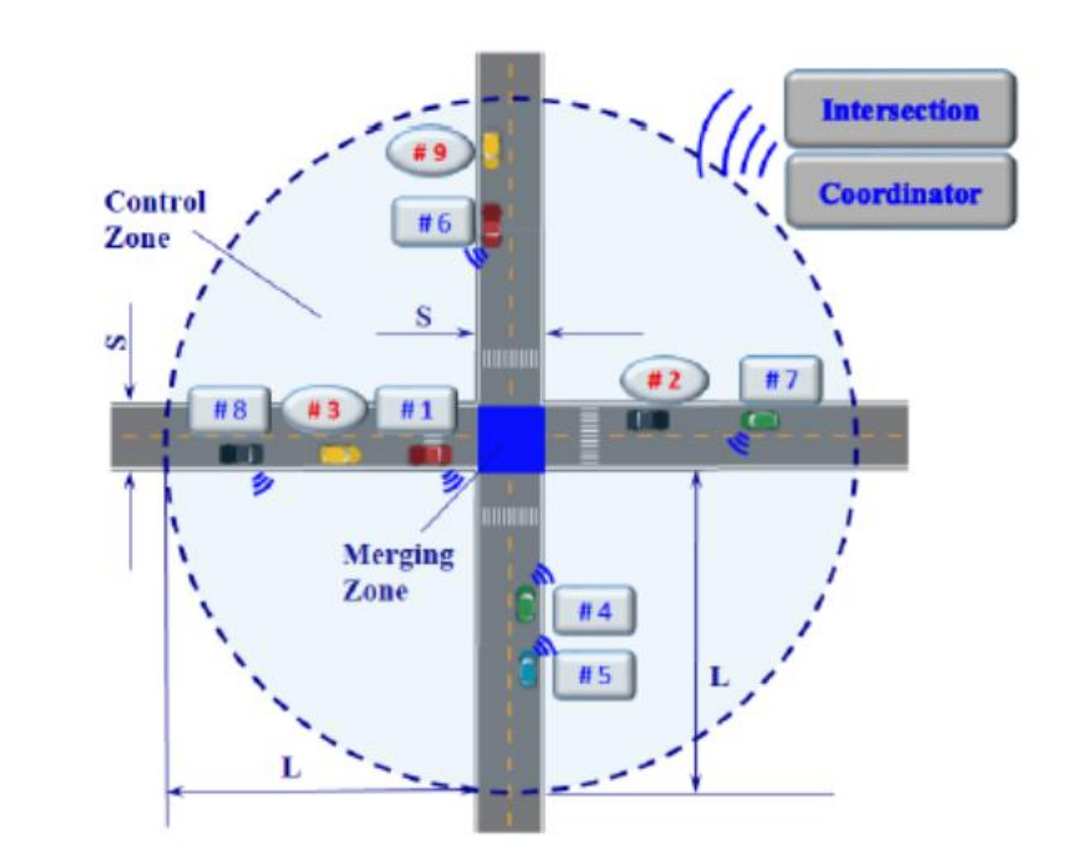
WIN-WIN !

+ fewer harmful emissions

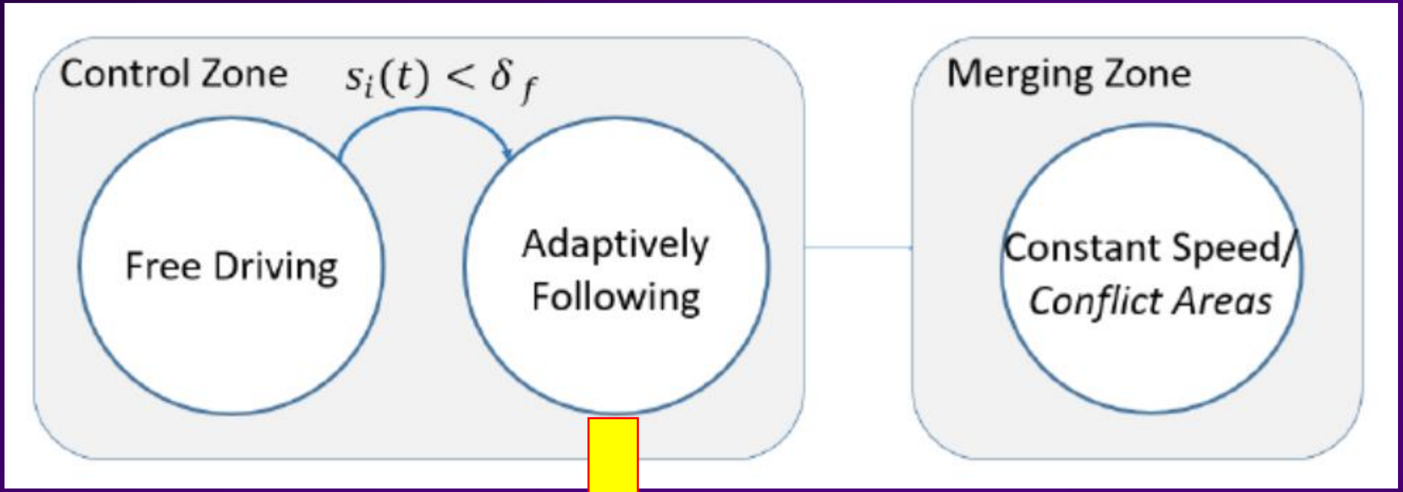


WHAT HAPPENS IN MIXED TRAFFIC ?

- **CAVs**
- **Non-CAVs**



MIXED TRAFFIC - *CAV BEHAVIOR*



$$\min_{u_i(t)} \frac{1}{2}\int_{t_i^0}^{t_i^m} [u_i^2(t) + \gamma(s_i(t) - \delta)^2]dt$$

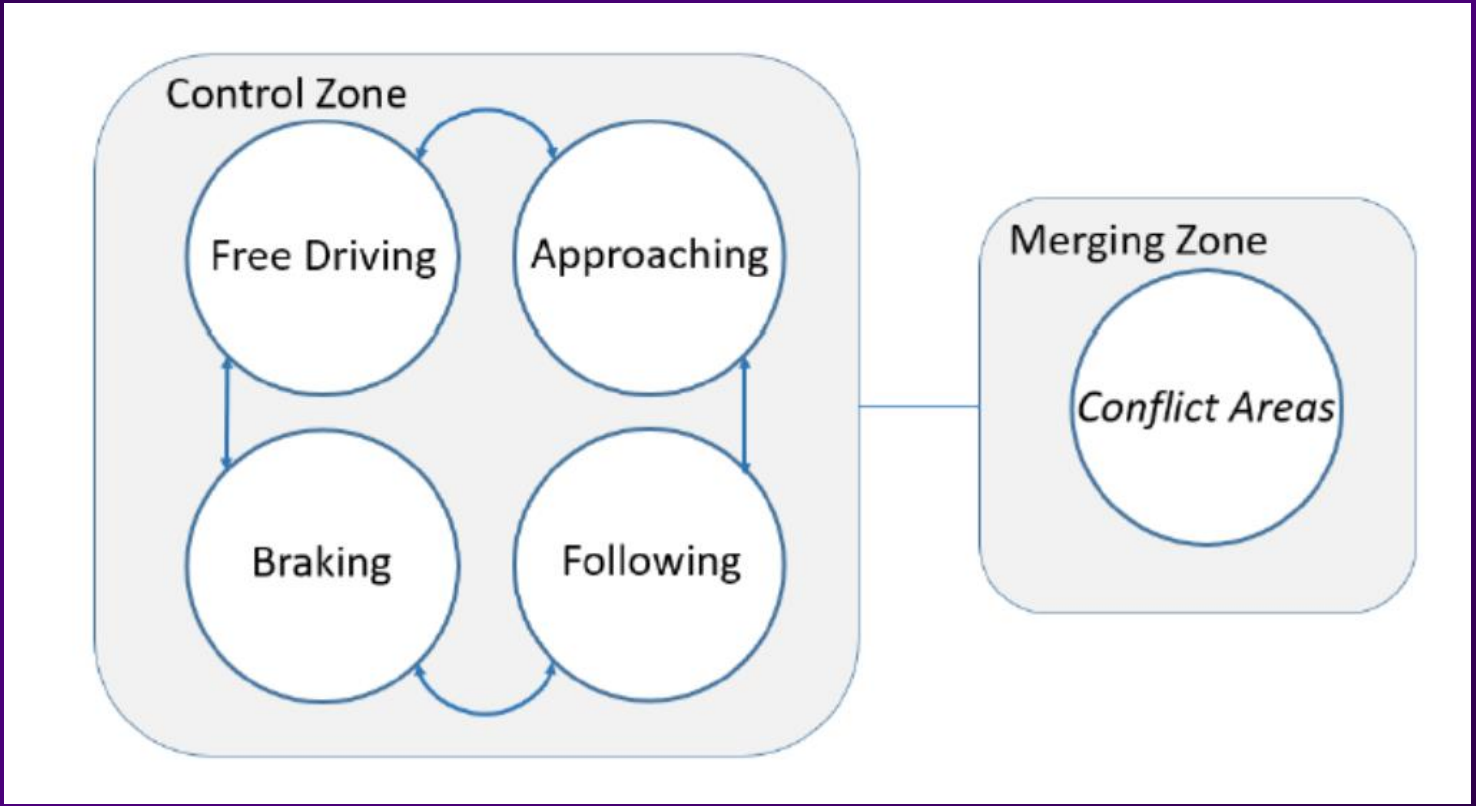
subject to : 1. CAV dynamics

2. Speed/Acceleration constraints

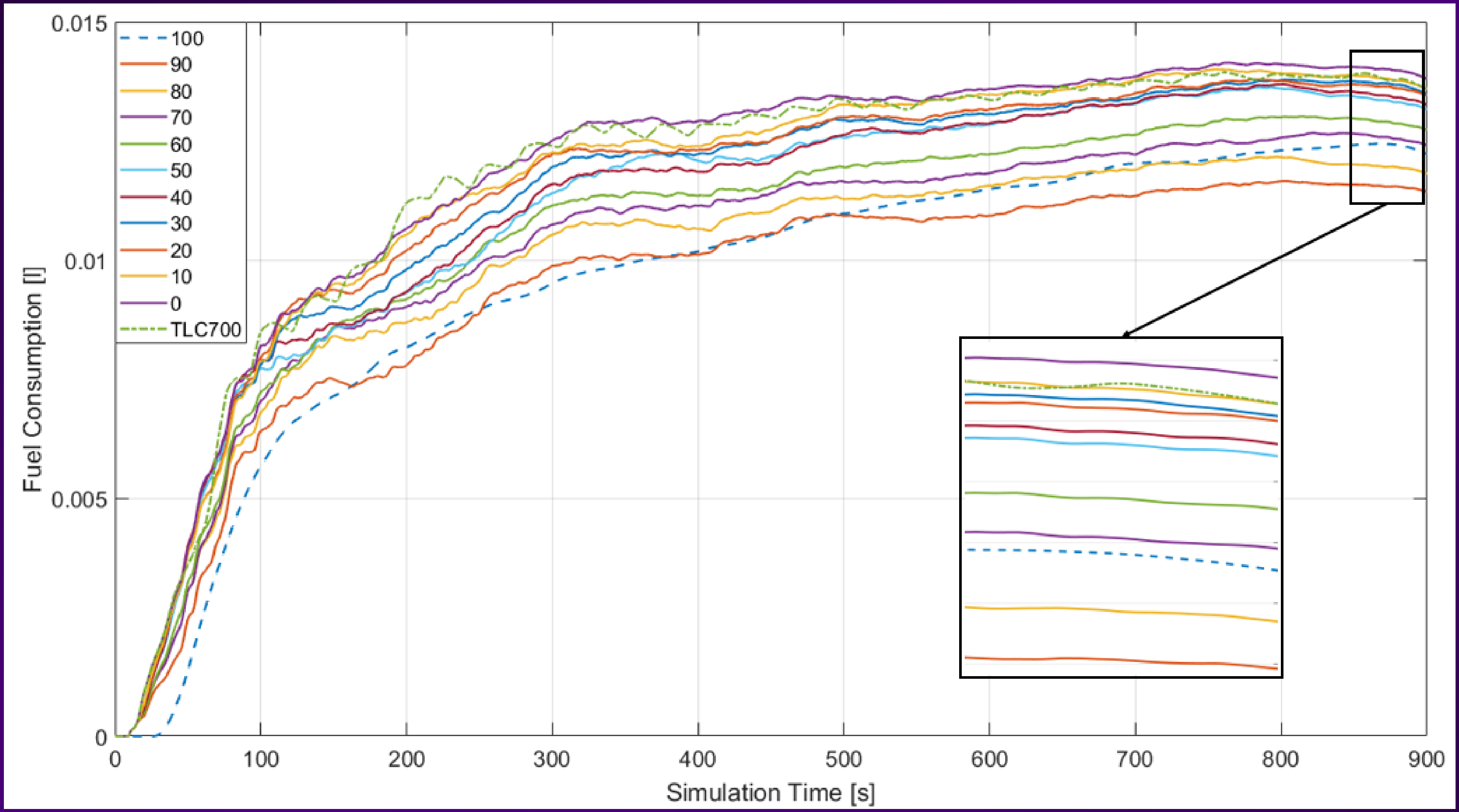
$t_i^m, \quad p_i(t_i^m) = L, \quad$ given : $t_i^0, \; p_i(t_i^0), \; v_i(t_i^0)$

MIXED TRAFFIC – *NON-CAV BEAVIOR*

- **Car-following behavior: The Wiedemann Model** ***[Wiedemann, 1974]***
- **Collision avoidance model in MZ through Conflict Areas.**

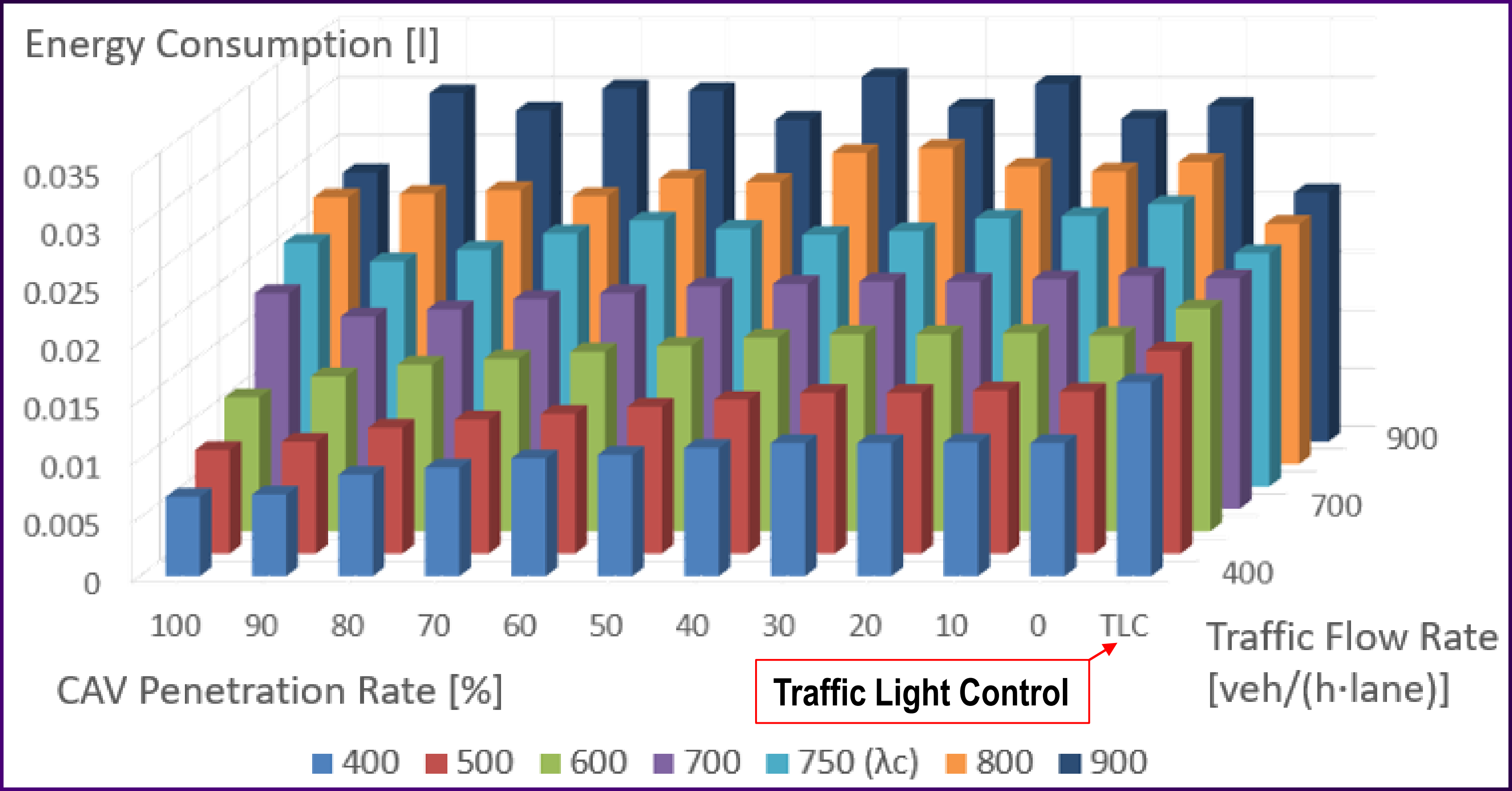


ENERGY IMPACT OF CAV PENETRATION



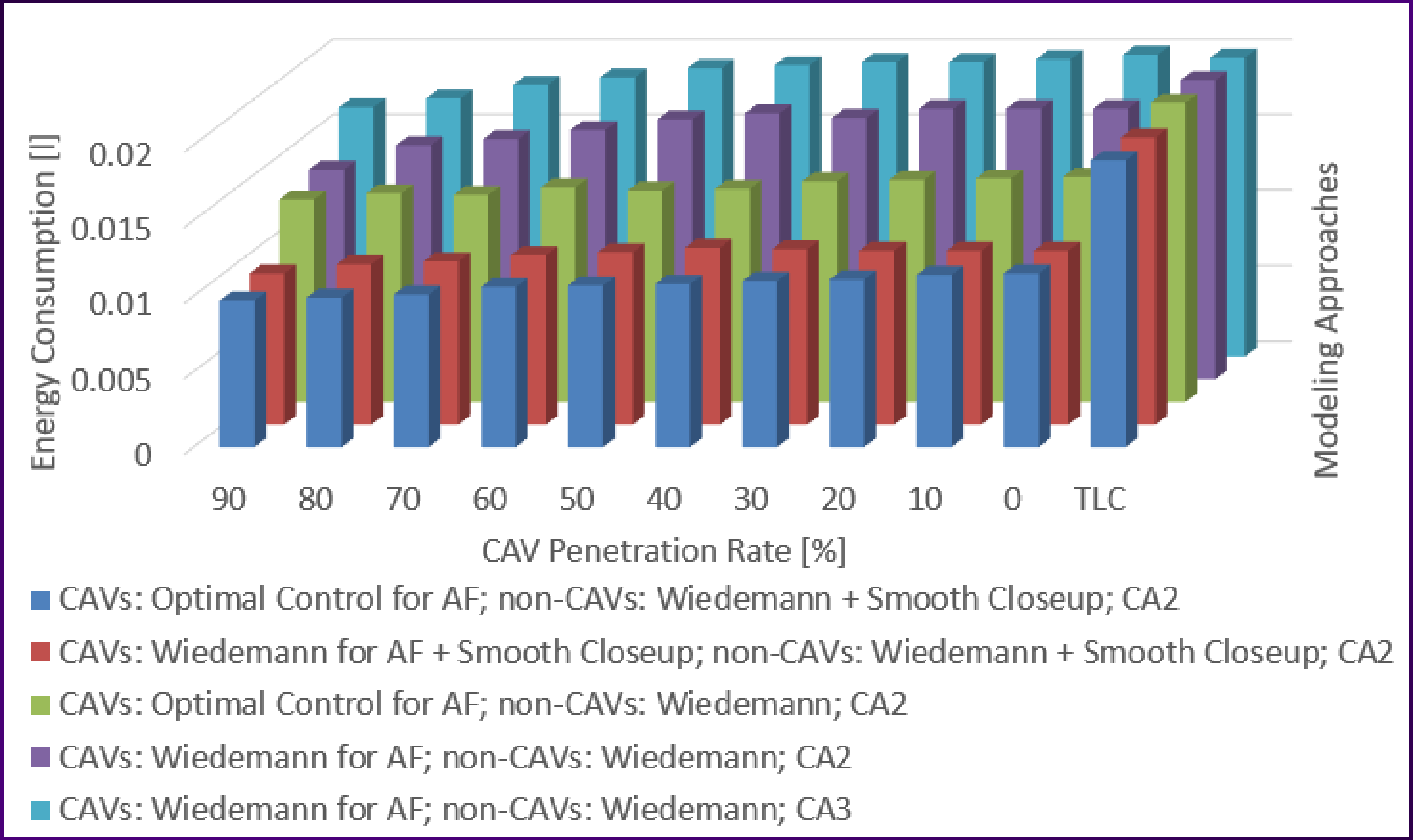
Traffic Flow Rate = 700 veh/(hourlane)

ENERGY IMPACT OF CAV PENETRATION



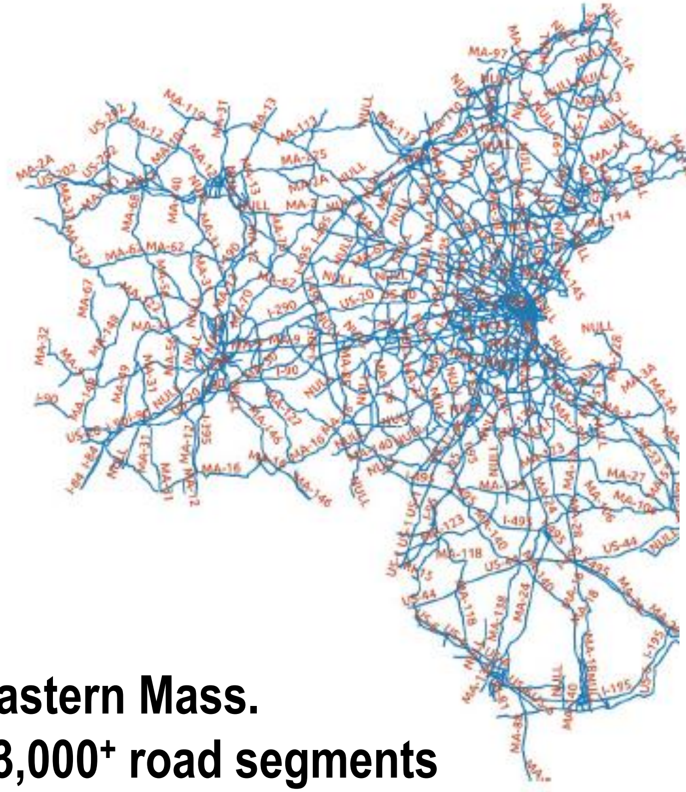
NOTE: Impact depends on Traffic Flow Rate !

ENERGY IMPACT OF CAV PENETRATION



NOTE: Impact depends on CAV and Non-CAV behavior models

CAV PENETRATION IMPACT IN TRAFFIC ROUTING

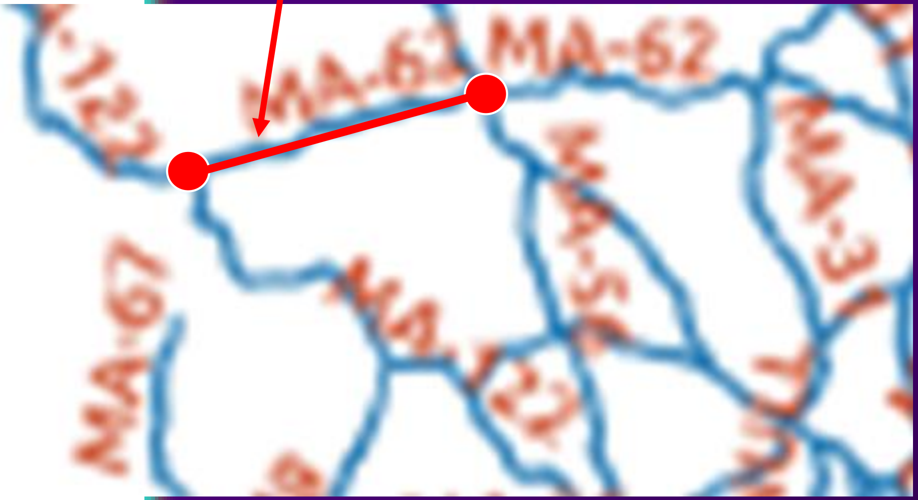


Eastern Mass.
13,000+ road segments

LINK a

FLOW x_a

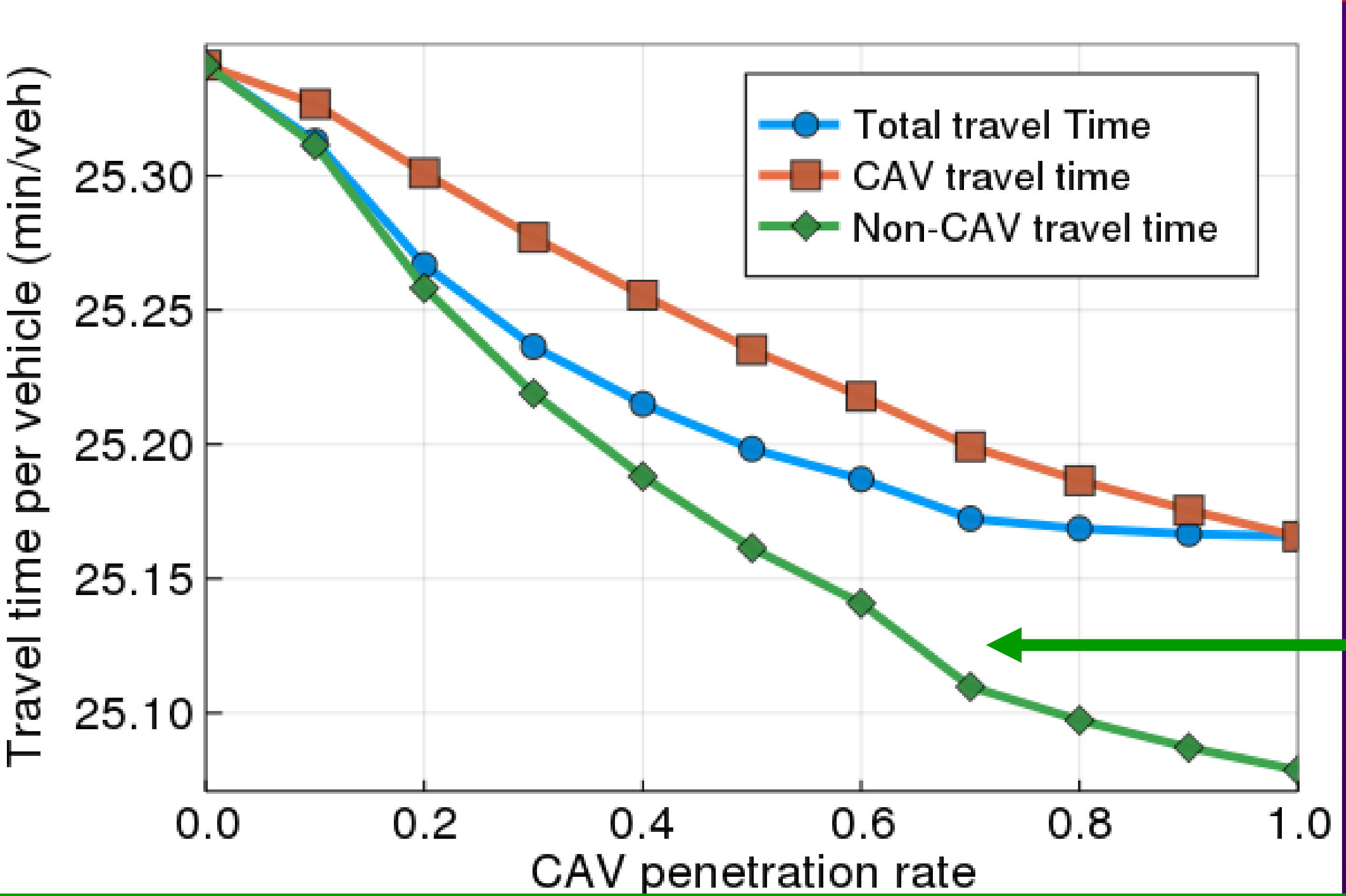
COST FUNCTION $t_a(x_a)$



USER-CENTRIC (selfish) control - Non-CAVs: x_a^{user} **s the equilibrium flow**

SYSTEM-CENTRIC (social) control - CAVs: x_a^{social} **is the equilibrium flow**

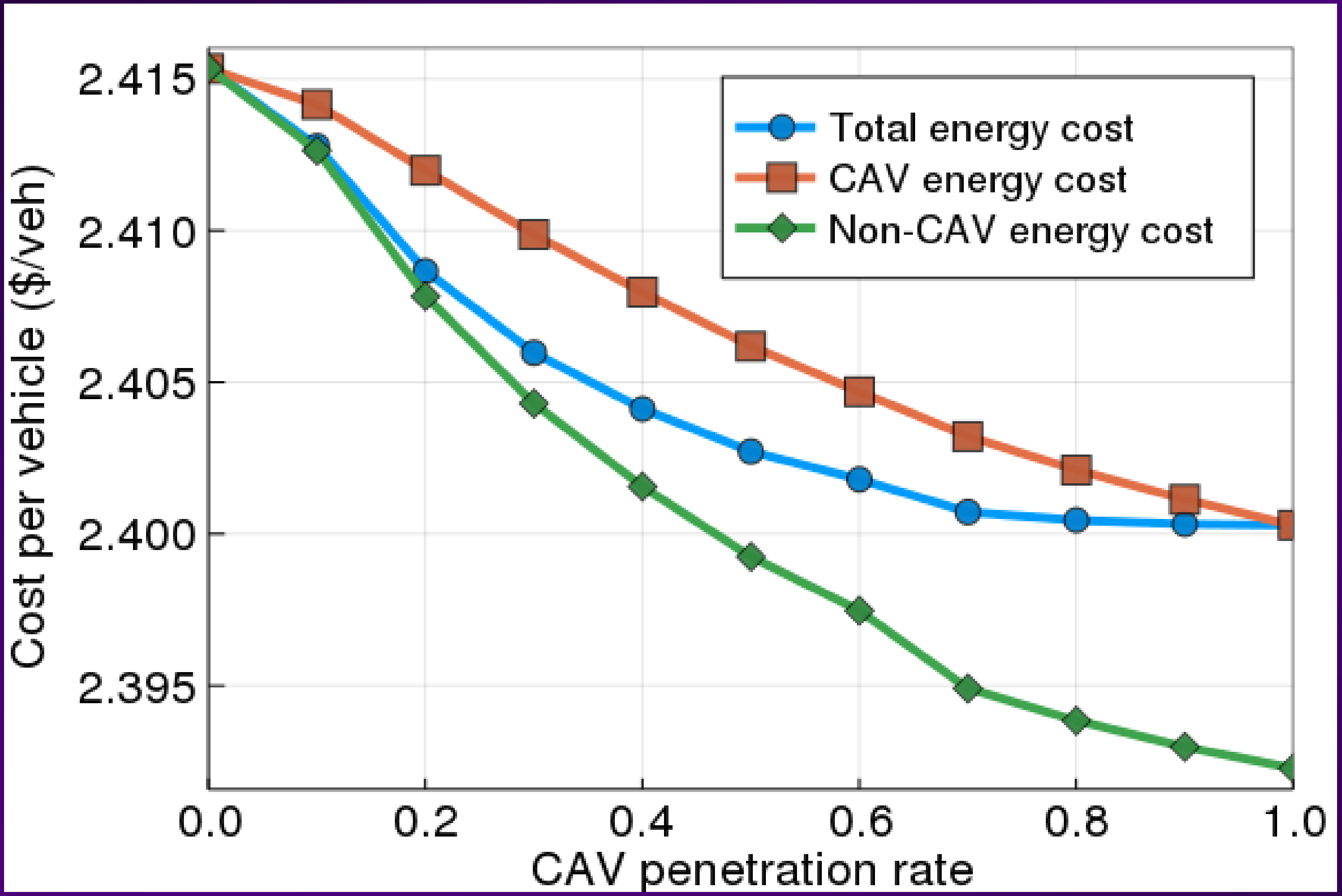
DO NON-CAVs BENEFIT FROM CAV PENETRATION?



Non-CAVs (selfish users) benefit from the addition of CAVs !

INTUITION: CAVs improve resource allocation for everyone, e.g., they decongest a link so that Non-CAVs still using this link benefit

DO NON-CAVs BENEFIT FROM CAV PENETRATION?



What incentive does a selfish user have to switch to a cooperative game setting (i.e., get a CAV) ???

CONCLUSIONS, OPEN QUESTIONS

- When it is optimal for CAVs to decelerate, Non-CAVs are induced to act optimally (natural platoons formed)
- When it is optimal for CAVs to accelerate, Non-CAVs become obstacles inducing sub-optimality
- Incentives for Non-CAVs to convert to CAVs ?
- Is *Shared* Mobility *On-Demand* the long-term answer ?
  (typical car utilization is 4%...)