Queues and Network Control for Urban Traffic Systems

Workshop on Control for Networked Transportation Systems
July 8 2019

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Overview

Transportation Science

Operations Research

Control Theory

Symbiosis between transportation and systems sciences
Tight integration essential for efficient use of data
Symbiosis between transportation and systems sciences
Symbiosis between transportation and systems sciences

Tight integration essential for efficient use of data
Transportation Queues

jobs → server

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Transportation Queues

- **mobility on demand**
  - jobs: pickup/delivery requests
  - server: vehicle fleet

- **signalized intersection**
  - jobs: vehicles
  - server: intersection

- **freeway (w/ CAVs)**
  - jobs: vehicles
  - server: freeway infrastructure

Service paradigms determined by automation and control.
Transportation Queues

Service paradigms determined by automation and control

- **mobility on demand**: jobs: pickup/delivery requests, server: vehicle fleet
- **signalized intersection**: jobs: vehicles, server: intersection
- **freeway (w/ CAVs)**: jobs: vehicles, server: freeway infrastructure
Performance Evaluation

\[ \lambda \rightarrow \text{server} \rightarrow \text{capacity? wait time?} \]

Constant Service Rate

\[ \lambda - c = \text{queue growth rate} \]

\[ \text{capacity} = c \]

Example: M/M/1

\[ c \equiv c(\text{queue length}) \]
Performance Evaluation

\[ \lambda \rightarrow \text{server} \rightarrow \text{capacity? wait time?} \]

Constant Service Rate

\[ \lambda - \frac{c}{\text{service rate}} = \text{queue growth rate} \]
Performance Evaluation

\[ \lambda \xrightarrow{\text{service rate}} \text{server} \xrightarrow{\text{capacity? wait time?}} \]

Constant Service Rate

\[ \lambda - c = \text{queue growth rate} \]

- capacity = c
Performance Evaluation

\[ \lambda \rightarrow \text{server} \rightarrow \text{capacity? wait time?} \]

Constant Service Rate

\[ \lambda - \overbrace{c}^{\text{service rate}} = \text{queue growth rate} \]

- capacity = \( c \)
- Example: M/M/1
Performance Evaluation

\[ \lambda - c = \text{queue growth rate} \]

- capacity \(= c\)
- Example: M/M/1

\[ c \equiv c(\text{queue length}) \]
State Dependent Transportation Queues

Diagram showing:
- Wait time vs Capacity
- Equation: $\Theta(\frac{\lambda}{m^2})$
- Equation: $\Theta(\frac{\lambda^2}{m^3})$

Outline of the Talk
- Network Flow
- State-dependent Queue
  - Flow conservation + Ohm (static)
  - Dynamic ($\dot{y}$)
  - Conservation law
- State-dependent service

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State Dependent Transportation Queues

[Diagram showing a graph with axes labeled: wait time vs capacity, and a variable λ on the x-axis.]

Outline of the Talk
Network Flow

State-dependent Queue

flow conservation + Ohm
(static)
?
y
(dynamic)
{ ... service
tra  cq u e u e
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State Dependent Transportation Queues

Diagram:
- Vertical axis: Wait time
- Horizontal axis: Capacity
- Lambda (\(\lambda\))

Outline of the Talk
- Network Flow
- State-dependent Queue
- Flow conservation + Ohm (static)
- Dynamic flow/mass conservation

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State Dependent Transportation Queues

wait time

capacity

overestimate

\[ \Theta(\frac{\lambda}{m^2}) \]

vs

\[ \Theta(\frac{\lambda^2}{m^3}) \]

vacation queue

0 100 200 300 400

Time

Average Queue Length

Vacation Queue

Webster Model

Vacation Queue (Time Average)

Uninterrupted Model

Akcelik Model

Outline of the Talk

Network Flow

State-dependent Queue

flow conservation + Ohm

(static)

?

(y)

(dynamic)

\{ ... service

tra
cq u e u e

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State Dependent Transportation Queues

\[ \Theta(\frac{\lambda}{m}) \] vs \[ \Theta(\frac{\lambda^2}{m^3}) \]

Outline of the Talk

Network Flow

State-dependent Queue

flow conservation + Ohm (static)

? (dynamic)

\{ flow, mass \}

conservation

state-dependent service

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State Dependent Transportation Queues

wait time underestimate capacity

\[ \Theta\left(\frac{\lambda}{m^2}\right) \]

\[ \Theta\left(\frac{\lambda^2}{m^3}\right) \]

processor sharing queue

Outline of the Talk
Network Flow

State-dependent Queue
flow conservation + Ohm
(static)
?
γ
(dynamic)
{ ... service
tra  cq u e u e
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State Dependent Transportation Queues

wait time vs capacity

\[ \Theta(\lambda/m^2) \]

\[ \Theta(\lambda^2/m^3) \]

spatial queue

Outline of the Talk

Network Flow

State-dependent Queue

flow conservation + Ohm
(static)

\( y \)

(dynamic)

{ ... service

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State Dependent Transportation Queues

wait time vs capacity

Spatial queue

\( \Theta(\lambda/m^2) \) vs \( \Theta(\lambda^2/m^3) \)

Vacation queue

Link 1, Link 2

Average Queue Length

0 100 200 300 400
0 2 4 6 8 10

Vacation Queue, Webster Model, Vacation Queue (Time Average), Uninterrupted Model, Akcelik Model
State Dependent Transportation Queues

wait time vs capacity

spatial queue

Θ(λ/m^2) vs Θ(λ^2/m^3)

vacation queue

Average Queue Length

vacation queue (Time Average)

processor sharing queue

Link 1

Link 2

λ

θ

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Current Notions of Capacity

Traffic Capacity [Highway Capacity Manual]

- “... maximum number of vehicles that can pass a given point ... (assuming) no influence from downstream traffic operation ...”
Traffic Capacity [Highway Capacity Manual]

- “... maximum number of vehicles that can pass a given point ... (assuming) no influence from downstream traffic operation ...”

- “... rate at which ... vehicles can traverse an intersection approach ... assuming that the green signal is available at all times ...”
Current Notions of Capacity

Traffic Capacity [Highway Capacity Manual]

- “… maximum number of vehicles that can pass a given point … (assuming) no influence from downstream traffic operation …”

- “… rate at which … vehicles can traverse an intersection approach … assuming that the green signal is available at all times …”

\[ f \leq c \]
Current Notions of Capacity

Traffic Capacity [Highway Capacity Manual]

- “... maximum number of vehicles that can pass a given point ... (assuming) no influence from downstream traffic operation ...”

- “... rate at which ... vehicles can traverse an intersection approach ... assuming that the green signal is available at all times ...”

\[ f \leq c \]

\[ c - f : \text{local robustness} \]
Current notions of capacity refer to the maximum number of vehicles that can pass a given point, assuming no influence from downstream traffic operation. This is also the rate at which vehicles can traverse an intersection approach, assuming that the green signal is available at all times.

Mathematically, this can be represented as:

\[ f \leq c \]

where \( f \) represents the flow and \( c \) represents the capacity. The difference \( c - f \) represents local robustness.
Towards Network Capacity

network capacity : ($\{c_i\}$, physical constraints, control)
Dynamical Network Flow

Mass Conservation

\[ \dot{x} = \lambda + R^T(x) f(x, u) - f(x, u) \]

- inflow
- outflow

\( x_i \): queue on link \( i \)

\( R(x) \): routing matrix

\( \lambda \)

\( \lambda_{\text{out}}(t) \)

\( f \)

\( i \)

\( x \)
Dynamical Network Flow

Mass Conservation

\[ \dot{x} = \lambda + R^T(x)f(x, u) - f(x, u) \]

- equilibrium \( x^\ast \): \( \lambda_{out}(t) = \lambda \)
- existence, stability, and robustness of \( x^\ast \)

\( x_i \): queue on link \( i \)
\( R(x) \): routing matrix
Distributed Feedback Control

\[ \min_{u} \int_{0}^{T} J(x(t), u(t)) \, dt \]

subj. to \[ \dot{x} = \text{traffic flow dynamics} \]

- \( u \equiv \text{ramp metering, variable speed limit, routing} \)
Distributed Feedback Control

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open-loop: \( u(t) \)
- exact convex relaxation
- distributed computation
Distributed Feedback Control

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- exact convex relaxation
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feedback: \( u(x) \) [ThC02.3]

- principled distributed control
- global computation of \( u(.) \)
Distributed Feedback Control

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\min_u \int_0^T J(x(t), u(t)) \, dt
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subj. to \( \dot{x} = \) traffic flow dynamics

- \( u \equiv \) ramp metering, variable speed limit, routing

**open-loop:** \( u(t) \)

- exact convex relaxation
- distributed computation

**feedback:** \( u(x) \) \([ThC02.3]\)

- principled distributed control
- global computation of \( u(.) \)
From State to Output Feedback Control

- direct access to $x$ not available
- $y$: detector measurement
From State to Output Feedback Control

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- “estimator” approach:
  $y \rightarrow \hat{x} \rightarrow u(\hat{x})$
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- “estimator” approach:
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- output feedback: $u(y)$
From State to Output Feedback Control

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- optimal output feedback traffic signal control
From State to Output Feedback Control

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- optimal output feedback traffic signal control
- pilot test: $\sim 20\%$ improvement w.r.t. incumbent
From State to Output Feedback Control

direct access to $x$ not available

$y$: detector measurement

“estimator” approach:

$y \rightarrow \hat{x} \rightarrow u(\hat{x})$

output feedback: $u(y)$

optimal output feedback traffic signal control

pilot test: $\sim 20\%$ improvement w.r.t. incumbent

Xtelligent
Concluding Remarks

- Transportation Science
- Operations Research
- Control Theory
- Queues
- Network Control

DATA

State-dependent queues, distributed/output feedback control for nonlinear systems...
Concluding Remarks

- state-dependent queues
- distributed/output feedback control for nonlinear systems
- ...

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